

The image cosines are basis vectors

The direction cosine data elements in Image Orientation (Patient) (20, 37) represent two of the three basis vectors that define the directions of the image slice axes. The first three values are the components of the image slice row vector

$$\mathbf{r} = [r_x \ r_y \ r_z].$$

The next three are the components of the image slice column vector

$$\mathbf{c} = [c_x \ c_y \ c_z].$$

The third basis vector of the image slice, the normal, is the cross product of the row and column vector

$$\mathbf{n} = [n_x \ n_y \ n_z] = \mathbf{r} \times \mathbf{c}.$$

The meaning of the image requirements

The direction cosines shall be normalized. The image axis basis vectors shall be unit vectors to within δ , on the order of the precision of floating-point arithmetic

$$\begin{aligned} |||\mathbf{r}|| - 1| &< \delta, \\ |||\mathbf{c}|| - 1| &< \delta, \\ |||\mathbf{n}|| - 1| &< \delta. \end{aligned}$$

The image axes shall be orthogonal

$$\begin{aligned} \mathbf{r} \cdot \mathbf{c} &= 0, \\ \mathbf{r} \cdot \mathbf{n} &= 0, \\ \mathbf{c} \cdot \mathbf{n} &= 0. \end{aligned}$$

The image axes shall be parallel or anti-parallel with respect to the axes of the patient coordinate system. Given the unit basis vectors of the axes of the patient coordinate system

$$\mathbf{i} = [1 \ 0 \ 0], \mathbf{j} = [0 \ 1 \ 0], \text{ and } \mathbf{k} = [0 \ 0 \ 1],$$

and a small deviation, ε , we require

$$|\mathbf{r} \cdot \mathbf{i}| > 1 - \varepsilon, \text{ and } |\mathbf{c} \cdot \mathbf{j}| > 1 - \varepsilon$$

for prone and supine patient orientations,

$$|\mathbf{r} \cdot \mathbf{j}| > 1 - \varepsilon, \text{ and } |\mathbf{c} \cdot \mathbf{i}| > 1 - \varepsilon$$

for decubitus patient orientations, and

$$|\mathbf{n} \cdot \mathbf{k}| > 1 - \varepsilon$$

in either case.